A-LEVEL
Mathematics
MPC3

UNIT: Pure Core 3

Mark scheme

6360

June 2017

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\left(\dfrac{dy}{dx}=\right)$ $A\sin 4x\dfrac{\sin 3x}{\cos^2 3x}+B\dfrac{\cos 4x}{\cos 3x}$ $A=3,\quad B=4$	M1 A1		$A\sin 4x\sec 3x\tan 3x+B\cos 4x\sec 3x$ $A, B\neq 0$
			2	
(b)	$\left(\int=\right)\ k\ln(2x^2+3)\qquad(+c)$ $\dfrac{3}{2}\ln(2x^2+3)+c$	M1 A1		Where k is a constant **Must have** +c as part of final answer
			2	
	Total		4	

Notes:

(a) Do not allow –(–3) for A1

Candidate using the quotient rule correctly, then **SC B1** for $\dfrac{4\cos 4x\cos 3x-(-3\sin 3x\sin 4x)}{\cos^2 3x}$ or better

(b) Condone poor use of brackets for M1 but only allow A1 if recovered.
Condone 6/4 for 3/2

<table>
<tr><th>Q2</th><th>Solution</th><th>Mark</th><th>Total</th><th>Comment</th></tr>
<tr><td>(a)</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>0.6</td><td>$e^{1.8-0.6^3} = 4.87441$</td></tr><tr><td>0.8</td><td>$e^{2.4-0.8^3} = 6.60614$</td></tr><tr><td>1</td><td>$e^2 = 7.38906$</td></tr><tr><td>1.2</td><td>$e^{3.6-1.2^3} = 6.50128$</td></tr><tr><td>1.4</td><td>$e^{4.2-1.4^3} = 4.28877$</td></tr></table></td><td>B1

M1</td><td></td><td>All 5 correct x values (and no extra used)
PI by 5 correct y values

At least 4 correct y values in exact form or decimals, rounded or truncated to 3 dp or better (in table or formula)
(**PI** by correct answer)</td></tr>
<tr><td></td><td>$\int = 0.2 \times (4.874.. + 6.606... + ...)$</td><td>dM1</td><td></td><td>Correct sub into formula with $h = 0.2$ **OE and 5** correct y values either listed, with + signs, or totalled.
(**PI** by correct answer)</td></tr>
<tr><td></td><td>$= 5.932$</td><td>A1</td><td></td><td>**CAO**, must see this value exactly and no error seen</td></tr>
<tr><td></td><td></td><td></td><td>4</td><td></td></tr>
<tr><td>(b)</td><td>$\left(\frac{dy}{dx} =\right)\ e^{3x-x^3}(3-3x^2)$ OE</td><td>B1</td><td></td><td>Do not condone poor use of brackets for this mark, unless written correctly later</td></tr>
<tr><td></td><td>Equating their $\frac{dy}{dx} = 0$</td><td>M1</td><td></td><td>**FT** 'their' $\frac{dy}{dx}$, PI by further working</td></tr>
<tr><td></td><td>$x = 1,\ y = e^2$
$x = -1,\ y = e^{-2}$</td><td>A1
A1</td><td></td><td>From $3 - 3x^2 = 0$ oe
And no 'extra' answers, coming from 'exponential terms'</td></tr>
<tr><td></td><td>$\frac{d^2y}{dx^2} = e^{3x-x^3}(-6x) + (3-3x^2)^2 e^{3x-x^3}$ OE</td><td>B1</td><td></td><td>Do not condone poor use of brackets for this mark, unless written correctly later</td></tr>
<tr><td></td><td>$x = 1,\ y''(1) = e^2(-6)$;
$x = -1,\ y''(-1) = e^{-2}(6)$</td><td>M1</td><td></td><td>Sub **both** correct x values into their $\frac{d^2y}{dx^2}$</td></tr>
<tr><td></td><td>$-6e^2 < 0$ so **maximum** at $x = 1$
$6e^{-2} > 0$ so **minimum** at $x = -1$</td><td>A1</td><td></td><td>Including inequalities (symbol or wording), and both conclusions.
Must have scored 2nd **B1**
Final **A** mark can be earned even if **A0A0** earlier</td></tr>
<tr><td></td><td></td><td></td><td>7</td><td></td></tr>
<tr><td></td><td>**Total**</td><td></td><td>11</td><td></td></tr>
</table>

Notes:

(b) May have used quotient rule to find $\frac{dy}{dx} = \frac{e^{x^3}3e^{3x} - e^{3x}3x^2e^{x^3}}{(e^{x^3})^2}$ for **B1**

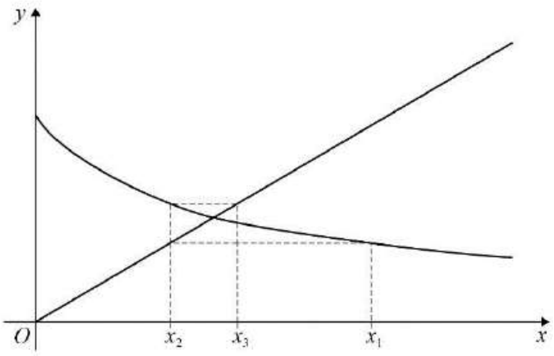
The values of the 2nd derivative may be evaluated (to –44.[3] and 0.8[12]), for the final **A1**
If a candidate has 3 'values' for x, then they lose the 2nd **A** mark, but subsequent marks are available

Q3	Solution	Mark	Total	Comment
	$\frac{\mathrm{d}u}{\mathrm{d}x} = -2\sin 2x$ OE	**B1**		
	$k\int u^2 \times (1-u^2)\,\mathrm{d}u$	**M1**		Condone omission of du
	$= m\int \left(u^2 - u^4\right)\mathrm{d}u$	**dM1**		Condone omission of brackets and du
	$= -\frac{1}{2}\left(\frac{u^3}{3} - \frac{u^5}{5}\right)$ $(+c)$ OE	**A1**		Must have seen du on an earlier line where all terms are in 'u' only
	$= \frac{\cos^5 2x}{10} - \frac{\cos^3 2x}{6}$ $(+c)$ OE	**A1**	**5**	Condone omission of $+c$
	Total		**5**	

Notes:

Withhold final **A1** for poor notation eg $\cos 2x^5$ but may have $\left(\cos 2x\right)^5$ for $\cos^5 2x$

Can score **A0A1** at end, if there is an omission of du

Q4	Solution	Mark	Total	Comment
(a)	$\mathrm{f}(x) = x - \ln\left(\dfrac{3x+10}{3x+1}\right)$ $\mathrm{f}(1) = -0.1(78...)$ $\mathrm{f}(2) = 1.(17...)$	**M1**		(or reverse) Both values rounded or truncated to at least 1sf
	Change of sign(or different signs) $\Rightarrow 1 < \alpha < 2$	**A1**		Must have both statement and interval in words or symbols **or** comparing 2 sides: at 1, $1 < \ln(13/4) = 1.1(79..)$; at 2, $2 > \ln(16/7) = 0.8(\ldots)$ (M1) Conclusion as before (A1)
			2	
(b)(i)	$x_2 = 0.827$ $x_3 = 1.277$	**B1** **B1**		Ignore further values
			2	
(ii)	 	**M1** **A1**		Vertical line from x_1 to the curve, seen or implied, and then horizontal to $y = x$ All correct with 2nd vertical and horizontal lines (only required above the '$y = x$' line), and x_2 , x_3 labelled on the x -axis
			2	
	Total		**6**	

Notes:

(a) Condone "less than or equal to"; allow "x" , "root" for α but not "it"

Candidates could change f(x) into exponentials eg $\mathrm{f}(x) = \mathrm{e}^x - \left(\dfrac{3x+10}{3x+1}\right)$ leading to f(1)=–0.5 and f(2)=5

Q5	Solution	Mark	Total	Comment
(a)	$x = \ln(3y+1)$ $e^x = 3y+1$ $[f^{-1}(x)] = \frac{1}{3}(e^x - 1)$ $[g(x) =]\frac{3}{3x+1}$	**M1** **M1** **A1** **B1**		Either order for **M1 M1**: Interchange x and y Correctly converting to e form. ACF
			4	
(b)	$\frac{3}{3x+1} = \frac{1}{3}(e^x - 1)$ $\frac{9}{3x+1} + 1 = e^x$ $\frac{3x+10}{3x+1} = e^x$ $x = \ln\left(\frac{3x+10}{3x+1}\right)$	**M1** **A1**		Correctly isolating term in e^x from 'their' $f^{-1}(x)$ and 'their' $g(x)$ Must see an intermediate line **AG** All correct and no errors seen
			2	
	Total		**6**	

Notes:

(a) Condone poor use of brackets if recovered

(b) Do not condone poor use of brackets even if recovered for **A1**

If a candidate has equation in terms of e^{x-1} then they must 'isolate x' correctly to score **M1**

Q6	Solution	Mark	Total	Comment
	$\int 3x(2x-1)^{-0.5}\ \mathrm{d}x$			
	$u=3x \qquad \frac{\mathrm{d}v}{(\mathrm{d}x)}=(2x-1)^{-0.5}$	**M1**		u and $\frac{\mathrm{d}v}{(\mathrm{d}x)}$ correct, with $\frac{\mathrm{d}u}{\mathrm{d}x}$ and $\int \mathrm{d}v$ attempted
	$\frac{\mathrm{d}u}{(\mathrm{d}x)}=3 \qquad v=\frac{2}{2}(2x-1)^{0.5}$	**A1**		All correct
	$\int = 3x(2x-1)^{0.5}-\int (2x-1)^{0.5}\times 3\ \ (\mathrm{d}x)$	**dM1**		Correct substitution of their terms into the parts formula
	$=3x(2x-1)^{0.5}-(2x-1)^{1.5}$ OE	**A1**		
	$[\int_1^5\ ]=(15\times 3-27)-(3-1)$	**dM1**		F(5) – F(1), correct from $Ax(2x-1)^{0.5}-B(2x-1)^{1.5}$
	$=16$	**A1**		
			6	
	Total		**6**	

Notes:
Check that an answer of 16 follows correct working

Q7	Solution	Mark	Total	Comment
	2 V-shaped mod graphs, one with vertex on positive x-axis and other with vertex on negative x-axis	**B1**		
	Critical values $\frac{15k}{2}$	**B1**		
	$5x-3k=-3(x+4k)$ OE	**M1**		PI
	$[x=]-\frac{9k}{8}$	**A1**		And no other values
	$x \leq -\frac{9k}{8}$, $x \geq \frac{15k}{2}$	**A1**	5	May have **OR** between two inequalities but not **AND**
	Total		**5**	

Notes:
For first **B1** condone line(s) extended, 'bending' to show intersection of lines
For **M1**, condone other symbols for '='
To find the cv's, a candidate might have squared and factorised, the **M1** is earned for $(8x+9k)(2x-15k)$, the accuracy marks are as above.
Mark last line as final answer

Q8	Solution	Mark	Total	Comment
(a)	$\tan^2 p = \sec^2 p - 1 \quad [= 11 - \sec p]$			
	$\sec^2 p - 1 = 11 - \sec p$	**M1**		Correct use of trig identity PI
	$\sec^2 p + \sec p - 12 = 0$			
	$(\sec p - 3)(\sec p + 4)[= 0]$	**A1**		Factorisation or correct use of formula PI
	$\sec p = 3, -4$	**A1**		Both correct and no errors seen
	$p = 1.23[...],\ 1.82[...],$	**B1**		Sight of **any** of these values correct to 2 dp
	$5.05[...],\ 4.459[...]$	**B1**		3 of these values correct to 2 dp
	$x = 0.88,\ 1.17,\ 2.49,\ 2.79$	**B1**		3 correct (must be to 2 dp)
		B1		All 4 correct (must be to 2 dp) and no extras in interval (ignore answers outside interval)
			7	
(b)	Stretch (I)			
	(Parallel to) x-axis (or line $y = 0$) (II)	**M1**		I and (II or III)
	SF 2 (III)	**A1**		I + II + III
	(followed by)			
	Translation through $\begin{bmatrix} k \\ 0 \end{bmatrix}$	**B1**		
	$\begin{bmatrix} -\dfrac{\pi}{6} \\ 0 \end{bmatrix}$	**B1**		
	OR			
	Translation through $\begin{bmatrix} k \\ 0 \end{bmatrix}$	**(B1)**		
	$\begin{bmatrix} -\dfrac{\pi}{12} \\ 0 \end{bmatrix}$	**(B1)**		
	(followed by)			
	Stretch (I)	**(M1)**		As above
	Parallel to x-axis (or line $y = 0$) (II)	**(A1)**		
	SF 2 (III)			
			4	
	Total		**11**	

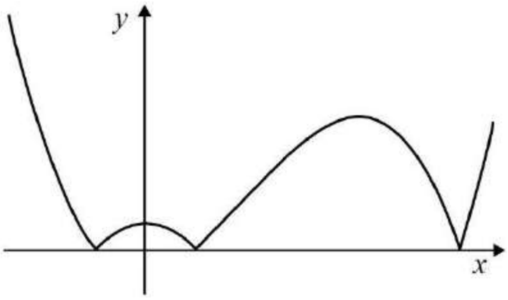
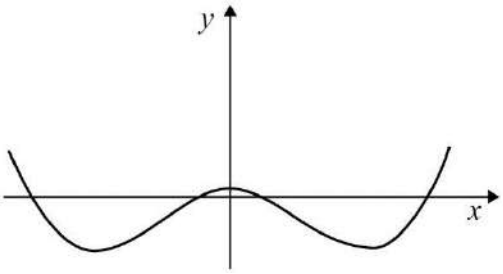
Notes:

(a) May use cos and sin leading to $\cos p = 1/3, -1/4$ for first **M1, A1, A1**

Condone using 'x' as 'p'

For first **B1** mark, this can be implied by one correct final value

For second **B1** mark, this can be implied by three correct final values, but do not accept values in terms of π

Q9	Solution	Mark	Total	Comment
(a)		M1 A1 A1		Modulus graph, 4 sections Correct on 2 ‘outside’ sections Correct on 2 ‘inside’ sections, 2 max, one on the *y*-axis (approx.), and correct cusps *Ignore any dotted sections*
			3	
(b)		M1 A1		Graph with exactly 1 max and 2 min All correct, symmetrical about *y*-axis
			2	
(c)(i)	$x = -a$ $y = 3b - 2$	B1 B1		Each value may be stated or given as coordinates
			2	
(ii)	$x = 0.5a$ $y = 27b$	B1 B1		Each value may be stated or given as coordinates
			2	
	Total		9	

Notes:

(b) The 2 min must be at the same ‘depth’ approx for **A1**

(c) Condone coordinates written in columns

(c)(i) Do not allow 0 – *a* for – *a*, nor *b* – 2 + 2*b* for 3*b*–2

Q10	Solution	Mark	Total	Comment
(a)(i)	$x = \ln 4, \quad y = e^{2\ln 4}$ $y = \left(e^{\ln 16} =\right)16$ $\frac{dy}{dx} = 2e^{2x}$ $y - 16 = 32(x - \ln 4)$ OE	 **B1** **M1** **A1**		 With no exponentials
			3	
(ii)	$[y=0] \quad -\frac{16}{32} = x - \ln 4$ $x = \ln 4 - \frac{1}{2}$ **or** $[x = \ln 4 - 0.5]$ $y - 16 = 32(\ln 4 - 0.5 - \ln 4)$ $y = 32 \times -0.5 + 16 = 0$	**B1**		Must see this line oe **AG** All correct and no errors seen. Must be using a correct equation from **(i)**, (condone $e^{2\ln 4}$ unsimplified)
			1	
(b)	$[\text{Cone} =] \frac{1}{3}\pi \times 16^2 \times (\ln 4 - (\ln 4 - 0.5))$ $= \frac{128}{3}\pi$ $[\text{For curve, Vol} =] \ \pi \int_0^{\ln 4} (e^{2x})^2 \ dx$ $[\int e^{4x} dx] = \frac{1}{4}e^{4x}$ $[\text{Vol} =] \ \pi \frac{1}{4}(e^{4\ln 4} - e^0)$ $= \pi \frac{1}{4}(256 - 1)$ $[= \frac{255}{4}\pi]$ $\text{Required 'vol'} = \left(\frac{255}{4} - \frac{128}{3}\right)\pi$ $= \frac{253}{12}\pi$	**M1** **A1** **B1** **M1** **dM1** **A1** **A1F** **A1**		**FT** "their" y=16 Correct including π, limits, dx Correct substitution of correct limits, including π (PI by next A1) Correct unsimplified exact value, no exponentials Vol = vol under curve – vol of cone, must have scored **M1M1**
			8	
	Total		**12**	

Notes:

(c) Condone poor use of brackets for **dM1**, if recovered
Condone omission of π for **dM1** if recovered